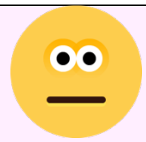
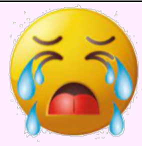
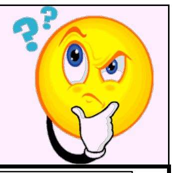
Vectors Topics

Two ways to define a line:
- If given a point and direction
- If given two points (subtract points for direction)

Ultimately, we just need to have a point and 1 direction and we can easily find the equation of a line in either vector or cartesian form

Four ways to define a plane:
- 3 points (not all on same line)
- Line and point not on line
- 2 distinct intersecting lines
- 2 parallel lines

Ultimately, we just need to have a point in a plane and a perpendicular direction

A Level Basics

Column vector representation				
Finding the magnitude of vector				
Finding the position vector and direction vectors				
Basic geometry including ratios and proving a straight line				
Finding vectors parallel to an axis or other vectors				
Basic angle calculations (includes between axes and within a triangle)				
Given magnitudes or angles, find unknowns				
x, y and z axis vectors				

Vector Equation Of A Straight Line

$$r = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = position, \begin{pmatrix} d \\ e \\ f \end{pmatrix} = direction\ (parallel\ to)$$

Given a point the line passes through and a parallel vector - find the vector equation of the line				
Given a point the line passes through and a line parallel to - find the vector equation of the line				
Given 2 points the line passes through - find the vector equation of the line				
Generating other points that lie on a line or generating parallel vectors when given a vector line equation				
3 Ways/Forms of writing a vector line equation 1. Position plus lambda times direction 2. Grouping the position and direction together $r = \begin{pmatrix} a+\lambda d \\ b+\lambda e \\ c+\lambda f \end{pmatrix}$ 3. Parametric $x = a+\lambda b$, $y = b+\lambda e$, $z = c+\lambda f$				
Given a vector line equation, find another line equation				
Understanding what vectors parallel to an axis mean: Parallel to x axis $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, Parallel to y axis $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, Parallel to z axis $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$				
Showing a point lies on a line				
Showing a point does not lie on a line				
Given a point lies on a line, find one or more unknowns				
Finding position vectors or unknowns based on knowing distances (this often comes up with circles and radii)				
Collinear – showing points are collinear or not collinear				
Given collinear and find unknown				

Cartesian Form Of A Straight Line

$$\frac{x-a}{d} = \frac{y-b}{e} = \frac{z-c}{f}$$

Given a point the line passes through and a parallel vector - find the cartesian equation of the line				
Given a point the line passes through and a line parallel to - find the cartesian equation of the line				
Given 2 points the line passes through - find the cartesian equation of the line				
Converting from vector line equation to cartesian line equation and vice versa				
Converting from vector cartesian equation to vector line equation and vice versa				

Vector Equation Of A Plane

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} d \\ e \\ f \end{pmatrix} + \mu \begin{pmatrix} r \\ s \\ t \end{pmatrix}$$

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = position, \begin{pmatrix} d \\ e \\ f \end{pmatrix}\ and\ \begin{pmatrix} r \\ s \\ t \end{pmatrix} = directions\ (parallel\ to\ the\ plane)$ and are not parallel to each other

Given 3 points - find the vector equation of a plane				
Given a contained line and another point - find the vector equation of a plane (Hint: Generate 2 points from the line)				
3 Ways/Forms of writing a **vector** plane equation 1. Position plus λ times direction plug μ times another direction 2. Grouping the position and direction together $r = \begin{pmatrix} a+\lambda d+\mu r \\ b+\lambda e+\mu s \\ c+\lambda f+\mu t \end{pmatrix}$ 3. Parametric $x = a+\lambda d+\mu r$, $y = b+\lambda e+\mu s$, $z = c+\lambda f+\mu t$				
Showing a point lies on a plane				
Showing a point does not lie on a plane				
Generating other points that lie on a plane or parallel vectors when given a vector plane equation				
Given a point lies on a plane, find one or more unknowns				
Showing 3 points are or are not coplanar				
Determining whether 4 points lie in the same plane				

Cartesian Equation Of A Plane

$$gx + hy + iz = d$$

$$d = distance\ form\ origin\ to\ plane, \begin{pmatrix} g \\ h \\ i \end{pmatrix} = direction\ vector\ (perpendicular\ to\ the\ plane\ this\ time)$$

Note: we plug point into $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ to get d

Given a point and perpendicular direction - find the cartesian equation of a plane				
Given 3 points - find the cartesian equation of a plane				
Given a line and a point not on a line - find the cartesian equation of a plane				
Given 2 parallel lines - find the cartesian equation of a plane				
Given 2 distinct intersecting lines - find the cartesian equation of a plane				
Showing a point lies on a plane				
Showing a point does not lie on a plane				
Given a point lies on a plane, find one or more unknowns				
Converting from vector plane equation to cartesian plane equation				
Converting from vector cartesian equation to vector plane equation				

Scalar Product

Properties and formula				
Parallel and perpendicular implications				
Using the formula to find angles and areas				
Finding the exact value of $cos\ \theta$				

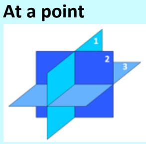
Finding a perpendicular vector

To another given vector				
To two given vectors				
To a line and another line (including the coordinate of the foot of perpendicular)				

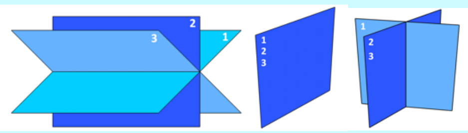
Intersections

Point and a line (occurs at a point)				
2 lines (occurs at a point or not at all)				
2 planes (occurs as a line or not at all)				
Line and plane (occurs at a point, line or not at all)				
3 planes (occurs at a point, line or not at all)				

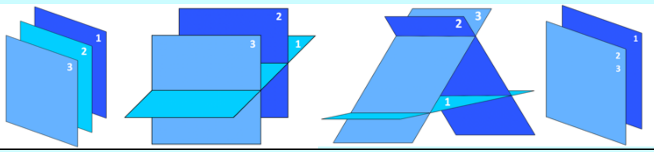
At a point



At a line



Not at all



Plane with axes				

Angle Between

2 vectors				
In a triangle				
2 lines				
Line and plane				
2 planes				
A line and its reflection				

Distances

A point and a line				
2 lines				
A point and a plane				
2 parallel planes				
A plane and a parallel line				

Reflections

Point in a line				
Point in a plane				
Line in a plane				

Vectors Formulae Sheet

Notations	$vector = \boldsymbol{a}, \underline{a}\ , \overrightarrow{\boldsymbol{OA}}$ distance= OA
Vector Form	$a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k} \equiv \begin{pmatrix}a\\b\\c\end{pmatrix}$
Properties (addition/subtraction, multiplication and scalar product)	$\begin{pmatrix}a\\b\\c\end{pmatrix} \pm \begin{pmatrix}d\\e\\f\end{pmatrix} = \begin{pmatrix}a \pm d\\b \pm e\\c \pm f\end{pmatrix}$ and $\lambda \begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}\lambda a\\\lambda b\\\lambda c\end{pmatrix} \begin{pmatrix}a\\b\\c\end{pmatrix} . \begin{pmatrix}d\\e\\f\end{pmatrix} = ad + be + cf$
Magnitude of a vector Notation is $\lvert\ \rvert$	$\left\lvert \begin{pmatrix}a\\b\\c\end{pmatrix} \right\rvert = \sqrt{a^2 + b^2 + c^2}$
Unit Vector	Unit vector of $\begin{pmatrix}a\\b\\c\end{pmatrix} = \frac{1}{\sqrt{a^2+b^2+c^2}} \begin{pmatrix}a\\b\\c\end{pmatrix}$
Parallel and Perpendicular to	Parallel means vectors are a multiple of each other Perpendicular means scalar product equals zero
Angle Between 2 vectors Always use the direction vectors	$\theta = \cos^{-1}\left(\frac{\begin{pmatrix}a\\b\\c\end{pmatrix} . \begin{pmatrix}d\\e\\f\end{pmatrix}}{\left\lvert \begin{pmatrix}a\\b\\c\end{pmatrix} \right\rvert \left\lvert \begin{pmatrix}d\\e\\f\end{pmatrix} \right\rvert} \right)$
Vector Equation of a line To find this we need: Point and direction (if given 2 points find the directions and use either point)	$r = \begin{pmatrix}a\\b\\c\end{pmatrix} + \lambda \begin{pmatrix}d\\e\\f\end{pmatrix}$ $\begin{pmatrix}a\\b\\c\end{pmatrix} = position, \begin{pmatrix}d\\e\\f\end{pmatrix} direction\ (parallel\ to)$
Cartesian Equation of a line	$\frac{x-a}{d} = \frac{y-b}{e} = \frac{z-c}{f}$ $\begin{pmatrix}a\\b\\c\end{pmatrix} = position, \begin{pmatrix}d\\e\\f\end{pmatrix} direction\ (parallel\ to)$
Parametric Form of a line	$x = a + \lambda d, y = b + \lambda e, z = c + \lambda f$
Equation of a plane	$\boldsymbol{r}.\boldsymbol{n} = \begin{pmatrix}a\\b\\c\end{pmatrix} . \boldsymbol{n}$ where n is the normal vector
Vector Equation of a plane To find this we need: a point in plane and perp direction. If not given perp direction take the cross product of 2 direction vectors. Remember to find a direction we subtract 2 position vectors	$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix} + \lambda \begin{pmatrix}d\\e\\f\end{pmatrix} + \mu \begin{pmatrix}r\\s\\t\end{pmatrix}$ $\begin{pmatrix}a\\b\\c\end{pmatrix} = position$ $\begin{pmatrix}d\\e\\f\end{pmatrix}\ and\ \begin{pmatrix}r\\s\\t\end{pmatrix} = directions\ (parallel\ to)$
Cartesian Equation of a plane	$ax + by + cz = d$ $d = distance\ form\ origin\ to\ plane$ $\begin{pmatrix}a\\b\\c\end{pmatrix} = direction\ vector\ (perpendicular\ to)$
Scalar Product Note: θ is the angle between $\begin{pmatrix}a\\b\\c\end{pmatrix}$ and $\begin{pmatrix}d\\e\\f\end{pmatrix}$	$\begin{pmatrix}a\\b\\c\end{pmatrix} . \begin{pmatrix}d\\e\\f\end{pmatrix} = \left\lvert \begin{pmatrix}a\\b\\c\end{pmatrix} \right\rvert \left\lvert \begin{pmatrix}d\\e\\f\end{pmatrix} \right\rvert cos\theta$
Vector Product Note: θ is the angle between $\begin{pmatrix}a\\b\\c\end{pmatrix}$ and $\begin{pmatrix}d\\e\\f\end{pmatrix}$	$\begin{pmatrix}a\\b\\c\end{pmatrix} \times \begin{pmatrix}d\\e\\f\end{pmatrix} = \begin{pmatrix}bf - ec\\-(af - cd)\\ae - bd\end{pmatrix}$ or $\left\lvert \begin{pmatrix}a\\b\\c\end{pmatrix} \times \begin{pmatrix}d\\e\\f\end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix}a\\b\\c\end{pmatrix} \right\rvert \left\lvert \begin{pmatrix}d\\e\\f\end{pmatrix} \right\rvert \sin\theta$
Area of a Parallelogram	$A = \left\lvert \begin{pmatrix}a\\b\\c\end{pmatrix} \times \begin{pmatrix}d\\e\\f\end{pmatrix} \right\rvert$ $\begin{pmatrix}a\\b\\c\end{pmatrix}$ and $\begin{pmatrix}d\\e\\f\end{pmatrix}$ form 2 adjacent sides of a parallelogram
Perp Distance between point and plane from (α, β, γ) to $ax + by + cz = d$	$\frac{\lvert\ a(\alpha) + b(\beta) + c(\gamma) + d \rvert}{\sqrt{a^2 + b^2 + c^2}}$
Scalar Product Properties	$0.\boldsymbol{a} = \boldsymbol{a}$ $\quad$ $\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{b}.\boldsymbol{a}$ $(-\boldsymbol{a}).\boldsymbol{b} = -(\boldsymbol{a}.\boldsymbol{b})$ $\quad$ $(k\boldsymbol{a}).\boldsymbol{b} = k(\boldsymbol{a}.\boldsymbol{b})$ $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a}.\boldsymbol{b} + \boldsymbol{a}.\boldsymbol{c}$ If a and b are parallel: $\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert$, moreover $\boldsymbol{a}.\boldsymbol{a} = \lvert\boldsymbol{a}\rvert^2$
Cross Product Properties (optional)	$\boldsymbol{a} \times \boldsymbol{a} = 0$ $\quad$ $\boldsymbol{a} \times 0 = 0 \times \boldsymbol{a} = 0$ $\lambda(\boldsymbol{a} \times \boldsymbol{b}) = (\lambda\boldsymbol{a}) \times \boldsymbol{b} = \boldsymbol{a} \times (\lambda\boldsymbol{b})$ $\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) + (\boldsymbol{a} \times \boldsymbol{c})$ $\boldsymbol{a} \times \boldsymbol{b} = -(\boldsymbol{b} \times \boldsymbol{a})$ $\boldsymbol{b}.(\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c}.(\boldsymbol{a} \times \boldsymbol{b})$